**Probability**

*Question: What is your understanding of probability?*

* Chance
* Uncertainty
* Likelihood

*Question: Give some examples of statements which has an element of probability in it*

* RCB might win the IPL this year
* The patient is probably suffering from heart disease
* The website may probably see 100 visits in the next 1 hour
* Suresh may score > 90 in the final exam
* Chances are high that diesel prices may go up again.

Embedded in the above statements is an element of uncertainty. What is probability then?

It’s a **numerical measure** of **uncertainty**. It is used in several fields including finance, medicine, weather forecasting etc.

**Terminologies**

*Question: If we toss a coin, what is the probability of getting a head?*

But before we get into more details around probability, let’s first talk about a ‘fair’ coin.

*Question: What is a fair coin?*

When we speak of a coin, we assume it to be ‘fair’, that is, it is symmetrical so that there is no reason for it to come down more often on one side than the other. In other words, it is an *unbiased* coin.

Question: What is a *random toss*?

By the phrase ‘random toss’, we mean that the coin is allowed to fall freely without any bias or interference. Random toss is an experiment where there is no bias on the part of the investigator.

Question: What are *Outcomes*? In a toss of coin what are the possible outcomes?

The possible outcomes depend on the experiment. If it is a single toss of coin the outcomes are {H, T). If it is two coins tossed simultaneously then the outcomes are {HH, HT, TH, TT}

Question: What is a *trial?*

A trial is the same as conducting an experiment. A toss of a coin is a trial. A roll of a die is a trial.

**Types of Probability**

*Question: Suppose you tossed a coin 20 times, how many would be Heads and Tails?*

*Question: Suppose you got 13 Heads and 7 Tails from random tosses of a fair coin? Why is it so?*

*Question: Do you observe equal probability in practise? What is the difference?*

There are two types of probability.

* Empirical Probability
* Theoretical Probability

*Empirical Probability*

Empirical Probability is also called experimental probability. It refers to probability observed from actual experiments. The formula is;

*Probability (Event) = Number of Trials in Which the Event Happened / Total No. of Trials*

Example:

A coin is tossed 1000 times with the following frequencies;

Head: 455 ; Tail: 545

Compute the probability for each event.

Probability(Head) = 455/1000

Probability(Tail) = 545/1000

*Theoretical Probability*

We know, in advance, that the coin can only land in one of two possible ways — either head up or tail up (we dismiss the possibility of its ‘landing’ on its edge, which may be possible, for example, if it falls on sand). We can reasonably assume that each outcome, head or tail, is as likely to occur as the other. We refer to this by saying that the outcomes head and tail, are equally likely.

The theoretical probability (also called classical probability) of an event E, written as P(E), is defined as;

*P(E) = Number of Outcomes Favourable to E/ Number of all possible outcomes of the experiment*

*Connection between Empirical & Theoretical Probabilities*

*Question: Are they always different? Under what conditions do they merge?*

When number of trials in an experiment is increased, the probabilities converge towards theoretical probabilities. When trials are repeated infinite number of times, both probabilities are equal.

**Example**

Two coins are tossed simultaneously 500 times, and we get

Two heads : 105 times

One head : 275 times

No head : 120 times

Find the probability of occurrence of these events?

*Question: Is this an empirical or theoretical probability in question?*

Prob (HH) = 105/500 = 0.21

Prob(HT or TH) = 275/500 = 0.55

Prob(TT) = 120/500 = 0.24

*What are the theoretical probabilities here?*

Possible Outcomes = {HH, HT, TH, TT}

Favourable Outcomes = {HH} = ¼ = 0.25; {HT, TH} = 2/4 = 0.5; {TT} = ¼ = 0.25

**Basic Relationships of Probability**

**Sample Space**

A sample space is the collection of all possible events or outcomes of an experiment. For example, there are two possible outcomes of a toss of a fair coin, which are a head and a tail.

**S = {H, T}**

**Event**

An event is an outcome or a set of outcomes of an activity or a result of a trial. For ex, getting two heads in the trial of tossing three fair coins simultaneously would be an event.

**Complement**

The complement of an event A is the collection of outcomes that are not contained in A. This complement is known as A ̒.

P[A] + P[A ̒] = P[S] = 1

P[A] = 1 – P[A ̒].

*Question: Why is complement required?*

We must have an intuition of which of the two probabilities are easier to compute. For instance out of 300 seats in an airplane, there are 13 empty seats, it is easier to count the empty seats to know how many are occupied.

**Mutually Exclusive Events**

Two events are said to be mutually exclusive if both events cannot occur at the same time as the outcome of a single experiment. For ex, toss of a coin, there cannot be both Heads and tails as outcome of a single toss.

**Addition Rule**

*(Independent / Mutually exclusive)*

When two events are mutually exclusive, then the probability that either of the events will occur is the sum of their separate probabilities.

For example, if you roll a single die, then the probability that it will come up with a face 5 or face 6 is given by

P(5 or 6) = P(5) + P(6) = 1/6 + 1/6 = 2/6 = 1/3

*(Not Independent / Not-mutually exclusive)*

If events A and B are not mutually exclusive, then the probability of occurrence of either event A or event B or both is equal to the probability that event A occurs plus the probability that event B occurs minus the probability that events common to both A and B occur.

P(A or B) = P(A) + P(B) – P(A and B)

**Venn Diagram**

1. Mutually exclusive events
2. Two non-mutually exclusive events
3. Three non-mutually exclusive events

**Example for non-independent event**

**Card Basics**

*Question: What is in a deck of cards?*

A deck of cards has 4 suits. Clubs, Diamonds, Hearts and Spades. There are 13 cards in each suit. 2 through 10, jack, queen, king, ace. So, there are 4 x 13 = 52 cards in the deck.

*What is the probability that a randomly drawn card is either an ace or a spade?*

Event A = An ace is drawn

Event B = A spade is drawn

Event [AB] = An ace of spade is drawn

P[A or B] = P(A) + P(B) – P(A and B)

= 4/52 + 13/52 – 1/52

=16/52 = 4/13